

HDP-003-001105

Seat No.

B. Sc. (Sem. 1) (CBCS) Examination

November / December - 2017

Mathematics: M-101

(Geometry & Calculus)

[Old Course]

Faculty Code: 003

Subject Code: 001105

Time : $2\frac{1}{2}$ Hours]

[Total Marks: 70

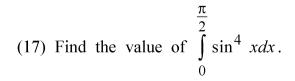
Instructions:

- (1) All questions are compulsory.
- (2) Right hand side figures indicate the marks.
- 1 Write the correct answer of following questions in short: 20
 - (1) Obtain the Cartesian co-ordinates for the polar co-ordinate $\left(1, \frac{\pi}{2}\right)$.
 - (2) Find the center of the circle $r^2 8r \cos\left(\theta \frac{\pi}{6}\right) + 12 = 0$.
 - (3) Write the general form for the equation of sphere.

- (4) Find the cylindrical form of the equation $x^2 + y^2 = 4$.
- (5) Write the expansion of $\cos x$ in terms of x.
- (6) Write the 3^{rd} term of series expansion of e^x .
- (7) Find y_7 , for $y = 3x^7$.
- (8) Find nth derivative of $y = xe^x$.
- (9) Define linear differential equation.
- (10) Write the necessary and sufficient condition for the differential equation to be exact.
- (11) Find the integrating factor of differential equation xdy + ydx = 0.
- (12) Write the order of differential equation

$$\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$$

- (13) Find $\frac{1}{D} x^2$.
- (14) Find the complementary function of the equation $\left(D^2 5D + 6 \right) y = 0.$
- (15) Find the complementary function of the equation $\left(D^2 2D + 1\right)y = 0.$
- (16) Write the expansion of $(1-D)^{-1}$.



(18) Find the value of
$$\int_{0}^{\pi/2} \cos^5 x dx$$
.

(19) Find the value of
$$\int_{0}^{\frac{\pi}{6}} \cos^{6} 3x dx$$
.

(20) Find the value of
$$\int_{0}^{\pi/2} \sin^4 x \cos^4 x dx$$
.

- 2 (a) Answer any three:
 - 1) Find the equation of line which passes through $A\left(4, \frac{2\pi}{3}\right)$ and perpendicular to OA.
 - (2) Convert the Cartesian equation $x^2 y^2 = a^2$ into the polar equation.

(3) If
$$y = e^{-3x} + e^{3x}$$
, then prove that $\frac{d^2y}{dx^2} = 9y$.

(4) Verify Roll's theorem for $f(x) = \cos x$, $x \in (0, 2\pi)$.

- (5) Using Maclaurin's series expand the function $f(x) = e^x$.
- (6) Evaluate $\lim_{x \to 0} \frac{\log x}{\cot x}$.
- (b) Answer any three:

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(1) Find center and radius of circle

$$r^2 - 4r \cos\left(\theta - \frac{\pi}{4}\right) = 5.$$

- (2) Find the equation of sphere passing through the circle $x^2 + y^2 + z^2 = 15$, 2x + 4y + 5z 7 = 0 and point (1, -1, 1).
- (3) Find the nth derivative of $\cos x \cos 2x \cos 3x$.
- (4) Verify Cauchy's mean value theorem for $f(x) = \sin x$,

$$g(x) = \cos x, \ x \in \left(0, \frac{\pi}{2}\right).$$

- (5) Expand $f(x) = \sin x$ in powers of $\left(x \frac{\pi}{2}\right)$.
- (6) Evaluate: $\lim_{x \to 0} \left(\frac{1}{e^x 1} \frac{1}{x} \right).$
- (c) Answer any two:

- (1) State and prove Leibnitz's theorem.
- (2) State and prove Lagrange's mean value theorem.

(3) Find the equation of tangent plane at any point $\left(\alpha,\beta,\gamma\right) \text{ of the sphere}$

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$
.

- (4) Prove that $\frac{x}{1+x^2} < \tan^{-1} x < x$, for x > 0.
- (5) Find the equations of the two tangent planes to the sphere $x^2 + y^2 + z^2 = 9$ which passes through the line x + y = 6, x 2z = 3.
- 3 (a) Answer any three:

- (1) Solve xdy + ydx = 0.
- (2) Solve $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0.$
- (3) Find the general solution of $p = \tan(px y)$.
- (4) Find the complementary function of $\left(D^2 5D + 6\right)y = 0.$
- (5) Find the particular integral of $(D-4)y = e^{2x}$.
- (6) Evaluate $\int_{0}^{\frac{\pi}{2}} \sin^4 x dx.$

(b) Answer any three:

(1) Solve
$$xy \frac{dy}{dx} = x^2 + y^2$$
.

(2) Solve
$$\frac{dy}{dx} + y \tan x = \frac{\cos x}{y}$$
.

(3) Solve
$$x^2 p^2 + pxy - 6y^2 = 0$$
.

(4) Solve
$$y = 2 px - \frac{p^2}{3}$$
.

- (5) Evaluate $\frac{1}{D^2}\cos 3x$.
- (6) Evaluate $\int \sin^5 x \cos^4 x dx$.
- (c) Answer any two:

- (1) Solve $(xy \sin xy + \cos xy) ydx + (xy \sin xy \cos xy) xdy = 0.$
- (2) Prove that $\frac{1}{f(D)}e^{ax}V = e^{ax} \frac{1}{f(D+a)}V$.
- (3) Obtain reduction formula for $\int \cos^n x dx$, $n \in N$.
- (4) State and prove necessary and sufficient condition for the differential equation Mdx + Ndy = 0 to be an exact equation.
- (5) Derive the formula to solve linear differential equation of first order and first degree.