HDP-003-001105

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Seat No. ________

B. Sc. (Sem. 1) (CBCS) Examination

November / December – 2017

Mathematics : M-101

(Geometry & Calculus)

[Old Course]

Faculty Code : 003

Subject Code : 001105

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Right hand side figures indicate the marks.

1 Write the correct answer of following questions in short : **20**

(1) Obtain the Cartesian co-ordinates for the polar co-ordinate $\left(1, \frac{\pi}{2}\right)$.

(2) Find the center of the circle $r^2 - 8r\cos\left(\theta - \frac{\pi}{6}\right) + 12 = 0$.

(3) Write the general form for the equation of sphere.

(4) Find the cylindrical form of the equation $x^2 + y^2 = 4$.

(5) Write the expansion of $\cos x$ in terms of x.

(6) Write the 3rd term of series expansion of e^x.

(7) Find y_7, for $y = 3x^7$.

(8) Find nth derivative of $y = xe^x$.

(9) Define linear differential equation.

(10) Write the necessary and sufficient condition for the differential equation to be exact.

(11) Find the integrating factor of differential equation $xdy + ydx = 0$.

(12) Write the order of differential equation

$$\frac{d^2 y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$$

(13) Find $\frac{1}{D} x^2$.

(14) Find the complementary function of the equation $\left(D^2 - 5D + 6\right) y = 0$.

(15) Find the complementary function of the equation $\left(D^2 - 2D + 1\right) y = 0$.

(16) Write the expansion of $(1 - D)^{-1}$.

(17) Find the value of $\int_0^{\frac{\pi}{2}} \sin^4 x dx$.

(18) Find the value of $\int_0^{\pi/2} \cos^5 x dx$.

(19) Find the value of $\int_0^{\pi/6} \cos^6 3x dx$.

(20) Find the value of $\int_0^{\pi/2} \sin^4 x \cos^4 x dx$.

2 (a) Answer any three : **6**

(1) Find the equation of line which passes through $A\left(4, \frac{2\pi}{3}\right)$ and perpendicular to OA.

(2) Convert the Cartesian equation $x^2 - y^2 = a^2$ into the polar equation.

(3) If $y = e^{-3x} + e^{3x}$, then prove that $\frac{d^2 y}{dx^2} = 9y$.

(4) Verify Roll's theorem for $f(x) = \cos x,\ x \in (0, 2\pi)$.

(5) Using Maclaurin's series expand the function $f(x) = e^x$.

(6) Evaluate $\lim_{x \to 0} \frac{\log x}{\cot x}$.

(b) Answer any three : **9**

(1) Find center and radius of circle

$r^2 - 4r\cos\left(\theta - \frac{\pi}{4}\right) = 5$.

(2) Find the equation of sphere passing through the circle $x^2 + y^2 + z^2 = 15$, $2x + 4y + 5z - 7 = 0$ and point $(1, -1, 1)$.

(3) Find the n^{th} derivative of $\cos x \cos 2x \cos 3x$.

(4) Verify Cauchy's mean value theorem for $f(x) = \sin x$,

$g(x) = \cos x$, $x \in \left(0, \frac{\pi}{2}\right)$.

(5) Expand $f(x) = \sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$.

(6) Evaluate : $\lim_{x \to 0} \left(\frac{1}{e^x - 1} - \frac{1}{x}\right)$.

(c) Answer any two : **10**

(1) State and prove Leibnitz's theorem.

(2) State and prove Lagrange's mean value theorem.

(3) Find the equation of tangent plane at any point (α, β, γ) of the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \cdot$$

(4) Prove that $\frac{x}{1+x^2} < \tan^{-1} x < x$, for $x > 0$.

(5) Find the equations of the two tangent planes to the sphere $x^2 + y^2 + z^2 = 9$ which passes through the line $x + y = 6,\ x - 2z = 3$.

3 (a) Answer any three : **6**

(1) Solve $xdy + ydx = 0$.

(2) Solve $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$.

(3) Find the general solution of $p = \tan(px - y)$.

(4) Find the complementary function of

$$\left(D^2 - 5D + 6\right) y = 0.$$

(5) Find the particular integral of $(D - 4)\, y = e^{2x}$.

(6) Evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x dx$.

(b) Answer any three : **9**

(1) Solve $xy\frac{dy}{dx} = x^2 + y^2$.

(2) Solve $\frac{dy}{dx} + y \tan x = \frac{\cos x}{y}$.

(3) Solve $x^2 p^2 + pxy - 6y^2 = 0$.

(4) Solve $y = 2px - \frac{p^2}{3}$.

(5) Evaluate $\frac{1}{D^2}\cos 3x$.

(6) Evaluate $\int \sin^5 x \cos^4 x dx$.

(c) Answer any two : **10**

(1) Solve

$(xy \sin xy + \cos xy)\, ydx + (xy \sin xy - \cos xy)\, xdy = 0$.

(2) Prove that $\frac{1}{f(D)} e^{ax} V = e^{ax} \frac{1}{f(D+a)} V$.

(3) Obtain reduction formula for $\int \cos^n x dx$, $n \in N$.

(4) State and prove necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be an exact equation.

(5) Derive the formula to solve linear differential equation of first order and first degree.